Calculation of hyperfine splitting in mesons using configuration interaction approach

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Abstract

The spin - spin mass splitting of light, heavy and mixed mesons are described within a good accuracy in the potential model with screened potential. We conclude that the long - distance part of the potential cannot be pure scalar and that a vector - scalar mixture is favoured. With the same parameters which gives correct average mass spectrum excellent spin - spin splittings of heavy quarkonia is obtained. The results are obtained by going beyond usually used perturbation method, namely using configuration interaction approach.

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1 Introduction

The problem of hyperfine splitting in mesons still attract wide interest. It is widely acceptable that quark potential model gives a rather good description of spin-average mass spectrum of hadrons , considered as composite system of quarks [1]. However, the question of explaining the influence of spin, namely spin-orbit („fine"), spin-spin („hyperfine") interaction, is not solved yet. The problem of mass splitting is due to spin structure and it is closely connected with the Lorentz - structure of the quark potential. These effects are far from being solved yet. Quite recently QCD motivated potential was applied to heavy quarkonia. Unfortunately authors of [2] restrited themselves to comparing the results to a single experimental value $J/\Psi - \eta_c$ mass difference. No attempt was made to calculate other quark-antiquark pairs. Spectroscopy of heavy mesons is studied also in paper [3]. The authors calculated only the spectra-mass of heavy mesons B, B_S, D, D_S with including spin-spin interaction. In [3] the first and second-order calculations of masses are in good agreement with the experimental data except for the higher spin states. Besides there are several works, in which the authors take limited number of pairs (like $u\overline{u}$) of particles for which splitting are calculated. Their results are quite good, but in work [4] authors limited themselves by considering only π, ρ, K, K^* mesons. In analogy Buchmuller and Tye [5] also considers only two systems, namely $c\overline{c}$ and $b\overline{b}$ though in states $1S$ and $2S$. They had obtaine good results for these particular pairs. There is another paper [6], in which both splitting and decay properties are studied. In this paper also good results for hyperfine splitting are obtained. But the authors calculate only few especial mesons ($J/\Psi - \eta_c$). Also the authors at calculations guess that in the hyperfine splitting gives contribution only the one - gluon term. From our paper it is visible, that confinement also plays an essential role at calculations the hyperfine effects in mesons.

The main problem of our work is to clarify some aspects of hyperfine interaction in the framework of configuration interaction approach (CI approximation, or CIA [7]).

Following many authors we assume admixture vector-scalar potential (soft model). We consider vector and scalar parts of static potential [1]

$$V(r) = V_S(r) + V_V(r) \tag{1}$$

where

$$V_V = -\frac{a_S}{r} + \varepsilon\frac{g^2}{6\pi\mu}\left(1 - e^{-\mu r}\right), V_S = (1-\varepsilon)\frac{g^2}{6\pi\mu}\left(1 - e^{-\mu r}\right) \tag{2}$$

and ε is the mixing constant.

The Hamiltonian can be written as

$$H = H_0 + H_{SS} \tag{3}$$

where for screened potential:

$$H_0 = \frac{1}{2m}\nabla^2 - \frac{a_S}{r} + \varepsilon\frac{g^2}{6\pi\mu}\left(1 - e^{-\mu r}\right) \tag{4}$$

where m- is reduced mass of $q\overline{q}$- system and, $h = c = 1$ units are used. In (3) only H_{SS} we take into account, because we calculate hyperfine splitting in S-waves. Then all terms in which contain the orbital quantum number l are absent.

In the frame of Breit-Fermi approach the spin-spin interaction term is

$$H_{SS} = \frac{2}{3m_{q_1}m_{q_2}}\overrightarrow{S_1}\overrightarrow{S_2}\Delta V_V \tag{5}$$

where $\overrightarrow{S}_{1,2}$ are the spin of the particles.

Now, we consider Shroedinger equation

$$\left(H_0 + H_{SS}\right)\Psi\left(\overrightarrow{r}\right) = E\Psi\left(\overrightarrow{r}\right) \tag{6}$$

Here we suggest to use CI approach, which was previously very successfully applied in atomic physics [7]. The essence of this approximation is that wave function (r) (r) is expanded in set of eigenfunctions of the Hamiltonian H_0, is

$$\Psi\left(\overrightarrow{r}\right) = \sum_n a_n\varphi_n\left(\overrightarrow{r}\right) \tag{7}$$

With substituting (7) in to (6) and using eigenvalue E_n^0 , we obtain homogeneous system of linear equations for a_n

$$a_n\left(E_m^0 - E\right) = -\sum_n a_n\left\langle\varphi_m\left|H_{SS}\right|\varphi_n\right\rangle \tag{8}$$

which have to be truncated for reasonable large n. In (8) is the eigenvalue of the nonperturbative Hamiltonian:

$$H_0\varphi_m = E_m^0\varphi_m \tag{9}$$

It is evident, that the solution of (8) exists only if the determinant, that contains of the coefficients are equal to zero. The diagonalization of this determinant gives the values of energies we are looking for. This is a good method to finding the eigenvalues. This procedure goes far outside of perturbative method.

2 Hyperfine splitting

In this work we obtained hyperfine splitting for heavy, light and mixed mesons using the configuration interaction approach. For example, we calculate the hyperfine splitting in and mesons with using oscillator potential.

Let us write down for this case Fermi-Breit equation for the two-quark system in the form

$$\left(-\frac{1}{2m}\Delta + Ar^2 - \frac{\alpha_S}{r} + H_{SS}\right)\Psi\left(\vec{r}\right) = E\Psi\left(\vec{r}\right) \tag{10}$$

where

$$H_0 = -\frac{1}{2m}\Delta + Ar^2 \tag{11}$$

$$H_{SS} = \frac{2}{3m_{q_1}m_{q_2}}\overrightarrow{S_1}\overrightarrow{S_2}\Delta V_V \tag{12}$$

and (12) is a additional for non-perturbatived Hamiltonian.

In this case vector and scalar part of the potential is equal accordingly:

$$V_V = -\frac{\alpha_S}{r} + \varepsilon A r^2 \text{ and } V_S = (1-\varepsilon)\, A r^2.$$

From one hand the oscillator potential not so bad in describing quarkonia data, on the other hand allowing to obtain analitic basic solutions. H_{SS} is the additional term, which have to be taken into account in CIA. Also one - gluon exchange term we put in to CIA. Below for obtaining the reliable results we shall use more realistic screened potential and make the numerical evaluation of the appropriate matrix elements. But for a while let us remain at using oscillator basis functions. Substituting the expanding the wave function into (10) and using the condition of ortonormality of the basic functions we obtain the algebraic system of equations for the coefficients a_n.

Two remarks are to be done in connection with (12). First we shall consider the contribution to spin-spin term of the potential consisting of both one-gluon and many-gluon exchanges (2). Therefore

$$H_{SS} = \frac{2}{3m_{q_1}m_{q_2}} \overrightarrow{S_1}\overrightarrow{S_2} \left(4\varepsilon\pi\alpha_S\delta\left(\overrightarrow{r}\right) + 6A\right) \tag{13}$$

$\overrightarrow{S_1}\overrightarrow{S_2} = -\frac{3}{4}$ for psevdoscalar mesons and $\overrightarrow{S_1}\overrightarrow{S_2} = \frac{1}{4}$ for vector mesons. In the latest calculations we shall take the interaction as a screened potential and we shall obtain the basic functions numerically and evaluate (6) with these functions. The second remark concerns to the meaning of parameter ε. According to general point of view [1] only vector contribution is present in (5). In order to avoid introducing additional parameters we shall consider the mixing parameter e to be same for many-gluon terms approximately equal to $\varepsilon = 0.5$.

If we restrict ourselves by one term in expansion (7) then (8) reduce to perturbation method. We present the wave function in (10) as two basic function of (7),

$$\Psi\left(\overrightarrow{r}\right) = a_1\varphi_1\left(\overrightarrow{r}\right) + a_2\varphi_2\left(\overrightarrow{r}\right), \tag{14}$$

immediately leads to a much better approximation. Namely one obtains for the energies

$$E_{1,2} = \frac{E_1^0 + E_2^0 + H_{11} + H_{22}}{2} \pm \frac{1}{2}\sqrt{\left(E_1^0 - E_2^0 + H_{11} + H_{22}\right)^2 + 4H_{12}H_{21}}, \tag{15}$$

where $H_{nm} = \langle \varphi_m \left| H_{SS} \right| \varphi_n \rangle$.

Let us write for completness the first two terms of oscillator wave-functions

$$\varphi_{1S}(r) = \left[\frac{2\beta^{3/4}}{\pi^{1/4}}\right] e^{-\frac{\beta r^2}{2}} Y_m^l$$

$$\varphi_{1S}(r) = \left[\frac{\sqrt{6}\beta^{3/4}}{\pi^{1/4}}\right] \left(1 - \frac{2}{3}\beta r^2\right) e^{-\frac{\beta r^2}{2}} Y_m^l$$

where $\beta = \sqrt{Am_q}$. In this case the corresponding matrix elements should be have the form

$$H_{11} = -\frac{2\alpha_S \beta^{1/2}}{\pi^{1/4}} + \frac{\overrightarrow{S_1}\overrightarrow{S_2}}{m_q^2}\left\{4A + \frac{8\alpha_S\beta^{3/2}}{3\pi^{1/2}}\right\}$$

$$H_{22} = -\frac{5\alpha_S \beta^{1/2}}{3\pi^{1/4}} + \frac{\overrightarrow{S_1}\overrightarrow{S_2}}{m_q^2}\left\{4A + \frac{4\alpha_S\beta^{3/2}}{\pi^{1/2}}\right\} \quad (16)$$

$$H_{12} = H_{21} = -\frac{2\alpha_S \beta^{1/2}}{\sqrt{6}\pi^{1/2}} + \frac{\overrightarrow{S_1}\overrightarrow{S_2}}{m_q^2}\left\{\frac{4\sqrt{6}\alpha_S\beta^{3/2}}{3\pi^{1/2}}\right\}$$

Substituting (16) into (15) and using standart values of E_m^0 for oscillator potential one can obtain the values for hyperfine splitting which are given in the Table 1. The parameters are chosen to be $A = 0.014$ GeV^3, $\varepsilon = 1$, $m_q = 1.5$ GeV, $\alpha_S = 0.32$ since exactly these parameters give the best results for oscillator potential in meson masses.

Table1.

	1	2	3	4	5	6	7	8	9	ΔM_{EXP}
$1S$	37	41	44	46	47	48	49	50	51	117
$2S$	–	39	43	46	48	49	51	52	53	95

Certainly using oscillator potential we obtain only didactic value. Therefore next we turned to use more realistic potential, namely, screened potential.The concrete form of such screened potential was previously suggested in [8]. We choose screened potential, because it gives excellent description of mass-spectrum in nonrelativistic potential models for heavy mesons. As Gerasimov pointed out [9] the QCD calculations on lattice indicates that the spin-spin forces are rather short range. Exactly the screened potential satisfies this condition. In this case even the basic solutions for unperturbed Hamiltonian can not be found in analitic form. We found these solutions numerically and evaluated the matrix elements numerically too. Final results for hyperfine splitting for screened potential are given in Tables 1,2. The following parameters were used in screened potential $\frac{g^2}{6\pi} = 0.224$ GeV^2, $\mu = 0.054$ GeV. All parameters was took from [10]. Experimental values were taken from [11].

Our calculations shown that the next terms of CIA method give contribution of order of 10% for heavy mesons and 35% for light mesons. Let us stress that the first term of CIA method is in fact just a perturbation result. CIA expansion better takes into account the interaction between particles. A similar approach was suggested in paper [12], where expansion was carried in basic function of oscillator potential. The suggested method is of considerable interest, since the perturbation method is still used as the practical method present [3], [4], [13].

3 Discussion

For reasons we present our data in 2 tables. We compare our results with results obtained in the works [12], [13], [14]. In the paper [12] good results are obtained, but the authors introduced additional parameters r_0. In the paper [13] hyperfine splitting is calculated in

the first account of perturbation theory. There are shown that the first order perturbations theory gives a good description of experimental data. The best results in paper [14] are obtained. But only one - gluon exchange is taken in spin-spin forces in the paper [14]. In all these works there is one common fault as in them are restricted to viewing limited of number mesons.

Table 2.

	[12] ΔM_{THEOR} (MeV)	[13] ΔM_{THEOR} (MeV)	[14] ΔM_{THEOR} (MeV)	Our results ΔM_{THEOR} (MeV)	ΔM_{EXP} (MeV)
$\Delta \mathrm{M}_{\rho-\pi}$	634	550	651	923	635
$\Delta \mathrm{M}_{\rho'-\pi'}$	329	-	-	411	150
$\Delta \mathrm{M}_{\varphi-\eta}$	217	-	270	580	320
$\Delta \mathrm{M}_{\varphi'-\eta'}$	135	-	-	285	-
$\Delta \mathrm{M}_{K^*-K}$	405	461	393	707	398
$\Delta \mathrm{M}_{K^{*\prime}-K'}$	195	-	-	336	200
$\Delta \mathrm{M}_{D^*-D}$	92	147	150	186	143
$\Delta \mathrm{M}_{D^{*\prime}-D'}$	-	-	-	112	-
$\Delta \mathrm{M}_{B^*-B}$	32	52	58	57	45.9
$\Delta \mathrm{M}_{B^{*\prime}-B'}$	-	-	-	36	-

Table 3.

	[12] ΔM_{THEOR} (MeV)	[13] ΔM_{THEOR} (MeV)	[14] ΔM_{THEOR} (MeV)	Our results ΔM_{THEOR} (MeV)	ΔM_{EXP} (MeV)
$\Delta \mathrm{M}_{D_S^*-D_S}$	87	190	128	163	144
$\Delta \mathrm{M}_{D_S^{*\prime}-D_S'}$	-	-	-	100	-
$\Delta \mathrm{M}_{B_S^*-B_S}$	-	-	-	50	47
$\Delta \mathrm{M}_{B_S^{*\prime}-B_S'}$	-	-	-	33	-
$\Delta \mathrm{M}_{B_c^*-B_c}$	-	-	-	49	-
$\Delta \mathrm{M}_{B_c^{*\prime}-B_c'}$	-	-	-	31	-
$\Delta \mathrm{M}_{\gamma-\eta_b}$	31	39	82	46	-
$\Delta \mathrm{M}_{\gamma'-\eta_b'}$	9	-	-	26	-
$\Delta \mathrm{M}_{\mathrm{J}/\Psi-\eta_c}$	65	100	112	110	117
$\Delta \mathrm{M}_{\Psi-\eta_c'}$	32	54	-	67	95

In Table 2 we show final data for light-quark systems, which has mainly relativistic character and compare them with other data.The calculation are carried out by using (1-9). In our quazirelativistic approach we should take into account in Hamiltonian also the term of order p^4 i. In paper [14] it was shown that the spectrum of Hamiltonian in nonrelativistic potential models, and spectrum of relativistic Hamiltonian for bound state and for first radial excited state are equivalent. On the other hand Lucha and Shoeberl [1] has shown that in some cases this relativistic kinematik term change the mass-spectrum drustically. But since we consider mass-difference therefore all relativistic effects must be cancel. In other papers authors introduce new additional parameters and obtain good description of hyperfine splitting. In paper [15] Faustov et al. also had

made similar calculations with using quazipotential. But evidently potential was choosen unsuccessfully, as ε turned out to be -0.9. Evidently this negative value of devoids it is clear physical sense, as a mixed parameters. Therefore this approach lost very much in heuristic understanding of obtained results.

In Table 3 we present the results of hyperfine splitting calculation in heavy-quark systems. Namely exactly for these systems our Breit-Fermi approach must be true with maximum extent. The obtained results for the hyperfine splittings of the S-wave states agree with measured splittings. As it is see most of our results have mainly predective character. In the Table 3 are shown that for $2S$-states we obtain somewhat worse results for hyperfine splitting than in $1S$-states. This can be the result of a mixing of S and D waves. In $2S$ state, as it is shown in work [16], the mixing can give contribution of 10%, while in case of $1S$-states the mixing correction is only of 1%. In other words we believe that taking into account the mixture of S and D waves, would considerable in proove our results.

We suggest that the potential consist of a sum of vector and scalar parts. This idea of scalar-vector mixing was discussed in [17], [18]-[20]. The authors of these papers also came to the conclussion that its mixing parameter must be different from zero. Franzinis [17] show, that VCONF must be tottaly scalar, while VOGE must be tottaly vector, but nevertheless they cannot give an adeqyate description of data, concerning the fine splitting. In series of papers [18]-[20] Deoghuria and Chakrabarty had choosen the confining potential in the form

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$$V = \varepsilon V_{OGE} + (1 - \varepsilon)\, V_{CONF}$$

and found $\varepsilon = 0.2$, treating as an adjustable parameter. In this paper we have used the same approach for CJP-type potential and found $\varepsilon = 0.5$. With this value of we described the hyperfine splitting of all mesons from heave to light ones.

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